

Worksheet 2-3: Exponential GrowthExponents in the Real World: Exponential Growth

Cell division is an example of exponential growth. Why? Let's find out.

Each one of you has a cell in your hand.

(Let's take out a sheet of paper and pretend it is the cell.)

Cell division is the process by which a cell divides into two cells. They are doubling!

(Let's start cutting the paper into halves!!)

Number of Cell Divisions	Number of Cells per Student Standard form Exponential form		Total Number of Cells in Class Standard form Exponential form	
Before cell division: 0				
After 1 st division: 1				
After 2 nd division: 2				
After 3 rd division: 3				
After n^{th} division: n				
After 8 th division: 8				
After 12 th division: 12				

Exponential growth can be modelled using an exponential function of the form:

$$y = ka^x,$$

where k is the initial amount,
 a is the change factor,
 and x is the number of changes over a given time.

In our investigation of cell division:

The initial amount of cells in the class: _____

The change factor: _____ is the _____ of the power.

The number of changes over time: _____ is the _____ of the power.

The formula to illustrate the exponential growth of total cells: _____

1. Some bacteria multiply by doubling every hour. You have 50 of this type of bacteria at the start. Find the number of bacteria you will have after 1 hour, 2 hours, 4 hours, and n hours.
2. Some insects multiply by tripling every day. You have 10 such insect at the start. Find the number of this insect you will have after 1 day, 3 days, 5 days, and n days.

3. The ancestors of Ling Ling, the giant panda, included 2 parents and 4 grandparents. Her grandparents were 2 generations before her.

(a) How many ancestors were in the seventh generation before Ling Ling? Express your answer in both exponential form and standard form.

(b) Explain your answer in part (a).

(c) How many ancestors were in the n^{th} generations before Ling Ling? Write a general expression in exponential form and standard form.

4. For the sequence: 1, 4, 16, 64, ...

(a) Describe the pattern in this sequence.

(b) Write the fifth and sixth numbers in the sequence in exponential form and standard form.

5. For the sequence: 1, 4, 27, 256, ...

(a) Describe the pattern in this sequence.

(b) Write the fifth and sixth numbers in the sequence in exponential form and standard form.

6. A scientist found that the number of bacteria in a culture doubled every hour. If there were 1000 bacteria at 08:00, how many were there at the following times?

(a) 09:00

(b) 14:00

(c) n hours later

Answers: 1. 100, 200, 800, $50(2^n)$; 2. 30, 270, 2430, $10(3^n)$; 3. (a) 2^7 , 128, (b) Initial amount is 1, Change factor is 2, Formula is 2^x , Find 2^x when $x = 7$, (c) 2^n ; 4. (a) Common ratio is 4 or Multiply 4 to the previous term to determine the next term in the sequence or The sequence is in the form 4^x starting at $x = 0$, (b) 4^4 , 256, 4^5 , 1024; 5. (a) The sequence is in the form x^x starting at $x = 1$ or Add 1 to both the base and the exponent of the power in the previous term to determine the next term, (b) 5^5 , 3125, 6^6 , 46656; 6. (a) 2000, (b) 64000, (c) $1000(2^n)$

Exponential Growth and Decay

$$\text{Total Amount} = \text{Initial Amount} \times (\text{factor of exponential change})^{\text{number of changes over time}}$$

Remember: **Exponential growth - the base is bigger than 1**
 Exponential decay - the base is between 0 and 1

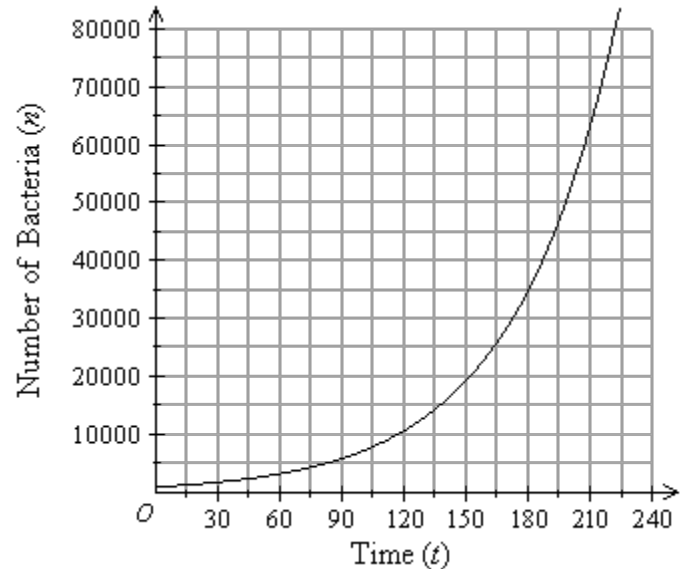
Examples:

- for an increase of 4% per year: base = $1 + 0.04 = 1.04$
- for an increase of 12% per year: base = $1 + 0.12 = 1.12$
- for a decrease of 15% per year: base = $1 - 0.15 = 0.85$
- for a decrease of 25% per year: base = $1 - 0.25 = 0.75$

Situation	Base	Equation	Result
\$1000 increases at 5% per year for 7 years.			
\$2000 increases at 8% per year for 15 years.			
Beginning at 1000 in 1990, the population increases 3% per year for 11 years.			
There are 50 bacteria in a dish at 12 noon. The number doubles every hour for 6 hours.			
A new car costs \$25 000. It depreciates at 12% per year for 4 years.			
There are 20 000 g of a radioactive substance. The amount is cut in half every month for 24 months.			
A dress sells regularly for \$150. Every month for 8 months, 5% is taken off the price.			
How many great, great, great grandparents does a person have?			
Starting with 16 000 litres of gas, Michel used 25% of it each week for 16 weeks.			
I like my new indoor grill so much, I told 5 friends about it this week. Each of them will tell 5 friends about it next week, and their friends will each tell 5 friends about it the week after that.			

1. The population of fish in a lake grows according to the expression $50(1.05)^t$, where t is the number of years.
 - a. Find the number of fish after 8 years, round to the nearest fish.
 - b. What is the annual growth rate for this fish population?
 - c. How many fish are there initially?

2. The population growth over time (measured in seconds) of a particular bacteria is displayed in the following graph.



- a. How many bacteria are alive after 1 minute?
 - b. How many bacteria are alive after 2 minutes?
 - c. How many bacteria are alive after 3 minutes?
 - d. How long did it take the population of this bacteria to reach 20000?
 - e. Initially, how many bacteria were there to start with?
3. The current population of the Earth is about 6 billion people. The population grows by about 1.4% per year. It is estimated that the Earth can support a maximum population of about 25 billion people. Determine how many years it will take to pass 25 billion. (Hint: Use your calculator.)
4. The deer population in a national park is declining every year. The population can be modelled using the formula $P = 380(0.975)^n$, where P is the population after n years.
 - a. What is the declining rate of the deer population? Write as a percent.
 - b. What is the expected deer population in 8 years, round to the nearest deer?

Answers: 1. (a) 74 fish, (b) 5%, (c) 50; 2. (a) 3000, (b) 11000, (c) 35000, (d) about 150 seconds, (e) about 1000; 3. 103 years; 4. (a) 2.5%, (b) 310