Worksheet 1-9: Ratios and Proportions

RATIOS:

> A **ratio** is a comparison of **like** quantities with the **same** units.
> **Equivalent** ratios have the **same simplest form** or the **same comparison.**

How many boys do we have in our class today? 20 Boys (1 quantity)
How many girls do we have in our class today? 5 Girls (1 quantity)

What is the ratio of number of boys to number of girls in the class?

Boys to Girls 20 : 5
= 4 : 1

What is the ratio of number of girls to number of boys in the class?

Girls to Boys 5 : 20
= 1 : 4

- No need to write units
- must be in lowest terms

Note the difference between the two ratios. **The order of the quantities is important.

Ratios can also be expressed in fraction form.

Practice:
For each of the following expressions:
a. Write the expression as a ratio.
b. Write the ratio in fraction form.

1. 4 dogs to 7 cats
(a) 4 : 7
(b) $\frac{4}{7}$

2. 5 pens to 11 books
(a) 5 : 11
(b) $\frac{5}{11}$

EQUIVALENT RATIOS:
Ratios are fractions, so we have to reduce ratios to lowest terms as well.
Write the following ratios in lowest terms.

3. (a) 4 : 12 ÷4 = 1 : 3
(b) 15 : 35 ÷5 = 3 : 7
(c) 3 : 9 : 27 ÷3 ÷3 ÷3 = 1 : 3 : 9

How many minutes are there in 1 hour? 60 min
How many cm are there in 1 m? 100 cm
Units are not written in ratios.
Convert to the smallest unit first
Write each ratio using the given information.

4. (a) 10 cm to 1 m ×100
= 10 : 100
= 1 : 10
(b) 4 min to 1 h ×60
= 4 : 60
= 1 : 15
(c) 15 s to 1 min ×60
= 15 : 60
= 1 : 4

PROPORTIONS:

> A **proportion** is a statement of **equality** between **equivalent** ratios.
> *Please note the equal sign between the ratios.

5. Find a ratio that is equivalent to the following ratios.
How?? *Either by dividing or multiplying both sides by the same factor!

(a) 2:5 ×4 ×4 = 8:20

(b) 8:6 ÷2 ÷2 = 4:3

(c) 1.5:3 ×10 ×10 = 15:30

(d) $9:\frac{1}{3}$ ×3 ×3 = 27:1

6. Write the above equivalent ratios as proportions.

(a) $2:5 = 8:20$

(b) $8:6 = 4:3$

(c) $1.5:3 = 15:30$

(d) $9:\frac{1}{3} = 27:1$

SOLVING PROPORTIONS FOR UNKNOWN QUANTITY:

7. Solve for the Unknown Value.

Two Terms:

(a) $x:3 = 1:6$

$\frac{x}{3} = \frac{1}{6}$

$6x = 3(1)$

$\frac{6x}{6} = \frac{3}{6}$

$x = \frac{1}{2}$

$x:3 = 1:6$ (÷2 ÷2)

$x = 1 \div 2$

$= \frac{1}{2}$

(b) $4:7 = 8:y$

$\frac{4}{7} = \frac{8}{y}$

$4y = 8 \times 7$

$\frac{4y}{4} = \frac{56}{4}$

$y = 14$

$4:7 = 8:y$ (×2 ×2)

$y = 7 \times 2$

$= 14$

Three Terms:

(c) $5:3:1 = 15:x:y$ (Hint: Break the 3-term proportion up into two 2-term proportions first.)

① $5:3 = 15:x$

$\frac{5}{3} = \frac{15}{x}$

$\frac{5x}{5} = \frac{45}{5}$

$x = 9$

$5:1 = 15:y$

$\frac{5}{1} = \frac{15}{y}$

$\frac{5y}{5} = \frac{15}{5}$

$y = 3$

② $5:3:1 = 15:x:y$ (×3 ×3, ×3)

$x = 3 \times 3$

$= 9$

$y = 1 \times 3$

$= 3$

Worksheet 1-10: Applications of Ratios and Proportions

Provide an answer statement for each of the following questions.
Identify variables used for each of the following questions.

RATIOS:

1. Kenny and Sara are business partners. Their shares in the business are 5 for Kenny to 3 for Sara. What is the ratio of Kenny's shares to Sara's shares to total shares?

Kenny : Sara : Total ← Labels
5 : 3 : 8
∴ The ratio is 5 : 3 : 8.

SOLVING PROPORTIONS:

TWO TERMS

2. One cup of uncooked rice yields 3 cups of cooked rice. How much uncooked rice is needed to yield 2 cups of cooked rice?

cooked : uncooked = c : uc
3 : 1 = 2 : x

$$\frac{3}{1} = \frac{2}{x}$$

$$\frac{3x}{3} = \frac{2}{3}$$

$$x = \frac{2}{3}$$

∴ $\frac{2}{3}$ cup of uncooked rice is needed.

THREE TERMS

3. A potting soil mixture contains 4 parts loam, 3 parts peat moss, and 1 part coarse sand. If there are 10 pails of loam to be used, how many pails of peat moss and of coarse sand are needed?

loam : peat : sand = l : p : s
4 : 3 : 1 = 10 : ? : ?

×2.5

Scale factor : $\frac{\text{Big \#}}{\text{Small \#}} = \frac{10}{4} = 2.5$

① $\frac{4}{10} = \frac{3}{p}$

$\frac{4p}{4} = \frac{30}{4}$

$p = 7.5$

$\frac{4}{10} = \frac{1}{s}$

$\frac{4s}{4} = \frac{10}{4}$

$s = 2.5$

② $p = 3 \times 2.5 = 7.5$

$s = 1 \times 2.5 = 2.5$

∴ 7.5 pails of peat moss and 2.5 pails of coarse sand are needed.

SOLVING PROPORTIONS WITH A TOTAL QUANTITY

4. Pewter is an alloy of 9 parts tin to 1 part copper. A pewter tray has a mass of 750g. What are the mass of tin and the mass of copper in the tray?

Tin : Copper : Pewter = t : c : P
9 : 1 : 10 = ? : ? : 750
(9+1=10; ×75 on each)

$\frac{\text{Pewter}}{\text{Tin}} : \frac{10}{9} = \frac{750}{t}$

$10t = 750 \times 9$

$\frac{10t}{10} = \frac{6750}{10}$

$t = 675$

$\frac{\text{Pewter}}{\text{copper}} : \frac{10}{1} = \frac{750}{c}$

$10c = 750 \times 1$

$\frac{10c}{10} = \frac{750}{10}$

$c = 75$

∴ There are 675 g of tin and 75 g of copper.

5. Alyssa, Bart, and Mickey are partners in a company in which they share all profits and losses in the ratio 5:3:2. In one year, the profits were $140,000. What is each partner's share of the profit?

Alyssa : Bart : Mickey : Total = a : b : m : Total
5 : 3 : 2 : 10 = ? : ? : ? : 140000

Method ①

Scale factor = $\frac{140000}{10} = 14000$ (× 14000)

Alyssa's share = 5 × 14000
= 70000

Bart's share = 3 × 14000
= 42000

Mickey's share = 28000

Check:
70000
42000
+ 28000
140000 → Total

Method ②

$\frac{\text{Alyssa}}{\text{Total}} : \frac{5}{10} = \frac{a}{140000}$

$\frac{700000}{10} = \frac{10a}{10}$

$70000 = a$

$\frac{\text{Bart}}{\text{Total}} : \frac{3}{10} = \frac{b}{140000}$

$\frac{420000}{10} = \frac{10b}{10}$

$42000 = b$

$\frac{\text{Mickey}}{\text{Total}} : \frac{2}{10} = \frac{m}{140000}$

$\frac{280000}{10} = \frac{10m}{10}$

$28000 = m$

∴ Alyssa's share is $70000, Bart's share is $42000 and Mickey's share is $28000.

12. **Take It Further** There are 900 students enrolled in Mount Forest Secondary School.
The ratio of girls to boys is 5 : 4.

a) How many boys and how many girls go to Mount Forest SS? Explain how you found your answer.

b) The average class size is 27 students. → total
Suppose this class is representative of all the students in the school.
How many students in this class are girls?
How many are boys?

g : b : total g : b : t
5 : 4 : 9 = ? : ? : 27
×3 ×3 ×3

$\frac{\text{girls}}{\text{total}}: \frac{5}{9} = \frac{x}{27}$

$\frac{9x}{9} = \frac{135}{9}$

$x = 15$

$\frac{\text{boys}}{\text{total}}: \frac{4}{9} = \frac{y}{27}$

$\frac{9y}{9} = \frac{108}{9}$

$y = 12$

8. Determine the value of each variable.

a) $4:10 = 18:c$ b) $125:25 = n:6$

① Scale factor $= \frac{18}{4}$
$= 4.5$

$C = 10 \times 4.5$
$= 45$

② Scale factor $= \frac{10}{4}$
$= 2.5$

$C = 18 \times 2.5$
$= 45$

③ $\frac{4}{10} = \frac{18}{c}$

$10c = 4 \times 18$

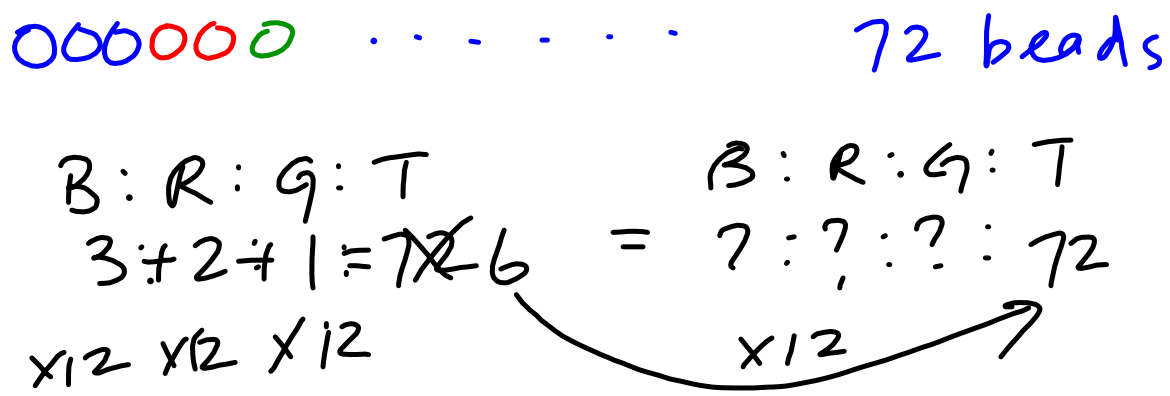
$\frac{10c}{10} = \frac{450}{10}$

$C = 45$

Oct. 4 Tuesday Unit 1 Test everything!

BEDMAS

variable substitution


72 beads
B : R : G : T
3 + 2 + 1 = 6
x12 x12 x12
B : R : G : T
? : ? : ? : 72
x12